Two economics facts about New York City's inclusionary housing program

February 13, 2016

This brief note summarizes two facts about the incentive structure created by the regulations in New York City's inclusionary housing program. First, the production of market-rate units with many bedrooms is very strongly discouraged. Second, the choice of how much affordable housing to require is a complex one.

1 A tax on bedrooms

Affordable units are governed by price ceilings and size standards. Perhaps out of all proportion to the drafters' intentions, the pivotal variable in this body of regulation turns out to be the number of bedrooms, whose importance emerges from four rules: (i) the collection of affordable units "shall contain a bedroom mix at least proportional to the bedroom mix" of market-rate units; (ii) there are size minima for each number of bedrooms in an affordable unit; (iii) the rationing system directs units with more bedrooms toward larger households; and (iv) the ceiling on the total rent—the product nsp—of an affordable unit depends on the size of its occupant household. The rent ceiling is 30% of what the U.S. Department of Housing and Urban Development calls "low-income" for each county and family size. And while New York City's IZ regulations do not explicitly stipulate a household size for each count of bedrooms, it is reasonable to assume the sort of rule in place in California[1]: a unit with n bedrooms goes to a household with $n+1$ members.

The ratios of the maximum total rents to the minimum area for each number of bedrooms yields a schedule $\hat{P}(n)$, which gives the rent per sq. ft. of affordable floorspace as a function of the number of bedrooms, n, in the market-rate units. The relevant figures for King County (coterminous with Brooklyn) appear in Table 1. These statistics work out such that $\tilde{P}(n) = .105 \cdot n^2 - .705 \cdot n + 2.94$ is exact (at least up to three bedrooms), although this formula was not consciously designed.

The takeaway is that $\tilde{P} < 0$: the developer is always better off with affordable units that have fewer bedrooms. Thus, if there is a function $P(n)$ giving the rent

[1] California State Code §50106(h)

n	max. income ($/year)[2]	max. rent ($/month)	min. size (sq.ft.)	$\hat{P}(n)$ ($)
0	47,700	1,175	400	2.94
1	53,700	1,342	575	2.33
2	60,400	1,510	775	1.95
3	67,100	1,678	950	1.76

Table 1: affordability parameters

on market-rate units as a function of the number of bedrooms, n, the developer is incentivized to choose a value of n somewhere below the global maximum. The effect resembles that of a tax on bedrooms. Past experience with this effect may explain one provision of New York's inclusionary housing ordinance: "not less than 50 percent of the...affordable housing units shall contain two or more bedrooms and not less than 75 percent...shall contain one or more bedrooms." Mayor de Blasio is very committed to creating affordable units with a large number of bedrooms: an article on de Blasio's bargaining over upzoning on the site of the former Domino Sugar Factory notes

> The de Blasio administration is not asking for more affordable units. It simply wants bigger ones — with fewer studios and one-bedrooms and more two- and three-bedroom units.

2 A Laffer Curve for affordable housing in New York City

In New York City, the density regulation, where a density bonus is available, is a ceiling on the Floor-Area Ratio (FAR): the quantity of floorspace that can be deployed per unit of land. Therefore, it will be convenient to specify the generic housing service, s, as sq. ft. of floorspace, and to assume that development takes place on a lot of unit size. With land fixed, the production function shall only have K as an argument, and the FAR ceiling is a maximum $\tilde{f}$ on $f(K)$, as in Bertaud and Brueckner (2005).

The density bonus policy consists of permitting an extra $1/\delta > 1$ sq. ft. of floorspace over $\tilde{f}$ for each sq. ft. the developer builds for affordable units. The offer is good up to a second-tier maximum $\tilde{f} \cdot (1+\psi)$. For example, in the R10 zoning district, the developer receives $1/\delta = 1.25$ extra sq. ft. for every one affordable, up to $\psi = 33\%$ over the standard FAR. It may be helpful to picture a 100% market-rate building of size $\hat{f}$ with a "bonus" section stacked on top that is $\delta \cdot 100\%$ affordable.

What constitutes an affordable unit? Here we will posit that there is a single size minimum and a price maximum for affordable units, leading to a maximum rent per sq. ft., $\tilde{p}$. (This ignores the bedroom considerations posed above for the sake of emphasizing the results presented here, which are not affected in-kind by making $\tilde{p}$ quasi-endogenous through the choice of bedrooms.) The developer's profit-maximization problem, then, is simply to choose the optimal building size:

$$\max_K \tilde{f} \cdot p + \left[f(K) - \tilde{f}\right] \{p \cdot (1-\delta) + \tilde{p}\delta\} - c_K K \text{ s.t. } f(K) \leq (1+\psi)\tilde{f}. \quad (1)$$

The first term in the objective gives revenue from the floorspace otherwise permitted by local zoning; the second revenue from bonus floorspace; the third variable costs. Now define

$$\bar{p} \equiv p \cdot (1-\delta) + \tilde{p}\delta$$

to be the average rent on bonus floorspace. Substituting $\bar{p}$ into 1 and taking the first-order condition on K yields

$$f\Big|_{K^*} = c_K / \bar{p}$$

as long as $f(K^*) \leq (1+\psi)\hat{f}$.

The quantity of construction, then, depends not on total profits, but on the average rent of bonus floorspace. Since $f'' < 0$, the FOC only holds for a unique value of K, which rises with $\bar{p}$. Inserting K^* into$f$, we obtain a function, $g(\bar{p})$ ($g' > 0, g'' < 0$), giving the profit-maximizing quantity of floorspace given $\bar{p}$.

A practical consequence is that a social planner maximizing the quantity of affordable floorspace—even one without regard for the developer's interests—may choose a $\delta < 1$. To see this, define the function $a(\delta)$ by the relation

$$a(\delta) = \delta \cdot \min\left\{g\left[\bar{p}(\delta)\right] - \tilde{f}, \psi f\right\}$$

to be the quantity of affordable floorspace provided conditional on δ. Since $\partial\bar{p}/\partial\delta < 0 \implies \partial g/\partial\delta < 0$, a is not guaranteed to rise for all $\delta \in [0,1]$.

Suppose, for instance, that $g(\tilde{p}) < \tilde{f}$—i.e., that the rent on an affordable unit is too low to justify a building of size $\tilde{f}$. In this case no affordable floorspace is built when $\delta = 1$ (when $\bar{p} = \tilde{p}$). But as long as $g(p) > \tilde{f}$, then for sufficiently low δ the developer will supply some affordable floorspace. Thus, we have $a(0) = a(1) = 0$ and $a'(0) > 0$, and there must exist some maximum value of δ in the interval $(0,1)$.

Another point worth making is that there is not *always* a trade-off between the total quantity of floorspace produced and the production of affordable floorspace. If $g(p) > \tilde{f}(1+\psi)$, then for small δ the developer will still supply $\tilde{f}(1+\psi)$ quantity of floorspace, making $a(\delta)$ linear at small values. Over some range the developer swaps market-rate for affordable floorspace at a one-to-one rate.